The Feedback Fubar

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Feedback is the most misunderstood topic in climate science and this misunderstanding extends to both sides of the debate. This is disturbing because the theoretical support for substantial warming cause by man's CO2 emissions depends exclusively on the ability of positive feedback to amplify something small (3.7 W/m^2 of forcing from doubling CO2) into something large (a 3C surface temperature rise).

What makes a 3C temperature increase relatively large compared to the forcing asserted to cause it is the difference between the equivalent emissions of a black body surface at the approximate average temperature of the planet (287K) and the equivalent emissions of a surface 3C warmer. Based on the Stefan-Boltzmann Law, the difference in equivalent emissions exceeds 16 W/m^2 and for that surface to emit this much more, it must be receiving an equal amount. Independent of the specific origin of the more than 12 W/m^2 of required surface input in excess of the 3.7 W/m^2 of input forcing, the amplification required exceeds a factor of 4. One thing that's clear is that the atmosphere has no internal sources of power, thus all of the power driving the 12 W/m^2 of additional surface emissions must be coming from feedback. This fact alone is sufficient to falsify claims of a high sensitivity since it's impossible for 3.7 W/m^2 of stimulus entering a passive system to result in more W/m^2 of feedback than it provides as input unless Conservation of Energy is violated. Climate science obfuscates this contradiction by considering only temperature and not the equivalent emissions of a temperature, thus a sensitivity of 0.8C per W/m^2 sounds plausible, yet in terms of joules, 4.3 W/m^2 of incremental surface emissions per W/m^2 of forcing does not, especially considering that each W/m^2 of incident solar energy results in only 1.6 W/m^2 of net surface emissions.

Theoretically, positive feedback can provide the required amplification, but only if the system being modeled conforms to the many assumptions that predicated Bode's control theory, originally developed in the 1940's as a tool for designing linear amplifiers using vacuum tubes.

Hansen was the first to apply Bode's analysis towards quantifying climate system feedback in his 1984 paper. Schlesinger quickly followed with a paper to 'correct' some of Hansen's errors but actually made it worse. This faulty analysis has been canonized by the IPCC since AR1 and the few related papers that followed simply restate Schlesinger's analysis using different variable names. An example is Roe's 2009 paper on climate feedback which will be referred to below.

While Bode's analysis provides the framework to achieve the required amplification, it can only do so under the specific conditions outlined in the first two paragraphs of his book. One of these conditions is the requirement for linear behavior between the input and output of the modeled network and another is the presence of linear vacuum tube elements with an implicit power supply that provides active gain which add energy to the output above and beyond what's supplied by the input stimulus.

It should be self evident that the Hansen/Schlesinger mapping to Bode violates both of these preconditions. First is that the input to the feedback network is forcing, expressed in W/m^2 while the output is in temperature, expressed in degrees K and that the relationship between W/m^2 and degrees K, as given by the Stefan-Boltzmann Law is very non linear. When the relationship between the input and output of an amplifier becomes non linear, Bode's formulation no longer applies and the gain becomes a function of the input rather than being strictly a function of the open loop gain and feedback. An example of this is when an audio amplifier starts to clip. The open loop gain and feedback remain constant, yet the closed loop gain steadily decreases as the input increases.

If a stimulus is applied to a complex, yet completely passive RLC circuit, all the nodes will wiggle, but this can never be considered equivalent to the behavior of an active system. Bode's assumption of active amplification is not relevant to the climate either. Many confuse the dynamics of weather as an indicator of an active system, but in the context of Bode, active and passive have very specific meanings. Passive means that there are no other sources of input other than the stimulus, which for the climate is the W/m^2 of forcing arriving from the Sun, while active means the system has powered gain driven by an implicit, internal power source. An important result of Bode's analysis is that a passive system is unconditionally stable which precludes the possibility of runaway positive feedback.

The difference between a passive system and an active system is like the difference between manual steering and power steering. Manual steering is a passive system that achieves force multiplication (gain) by a combinations of gears, levers and pivots as energy is conserved between the steering wheel and the tires. Power steering is an active system that positively reinforces arm muscles by adding energy to the system from a hydraulic power source driven by the engine. The climate is a passive system that manifests surface temperature amplification by delaying surface emissions and returning them to the surface some time in the future where they are combined with new incident power from the Sun. It's these joules of energy being delayed and returned back to the surface that comprises the physical manifestation of climate system feedback. This feedback is tangible, which for the climate is expressed in W/m^2 which are added to the new input from the Sun also quantified as W/m^2. Watts are joules per second and first principles requires joules to be conserved.

Bode's feedback model removes the requirement of Conservation of Energy between the input and output of the system. This is the result of assuming an external power supply will provide as much output as required. This isn't valid for a passive system like the climate where solar input is the only source of power and thus COE must be accounted for. Unlike an active amplifier which measures the input and feedback to determine how much output to deliver from an unlimited source, a passive system consumes its input and feedback to produce its output. Disconnecting the input and output from the requirements of COE makes sensitivities that violate COE seem plausible and this is the only reason that such an unreasonably high sensitivity can be accepted. When COE is added to the analysis, the maximum possible sensitivity becomes less than the lower limit claimed by the IPCC.

Technically speaking, the model proposed by Hansen called the system input a change in forcing and its output a change in surface temperature. For the linear systems modeled by Bode's equations, the absolute and incremental gain are the same and independent of the magnitude of the input or output. For the climate system feedback model, this is an invalid assumption owing to the non linearity between W/m^2 of input and degrees K of output, where the ratio of a change in output temperature per change in input forcing depends on the starting temperature. To get around this, it's asserted that the system is approximately linear, but the feedback formulation sets the reference temperature to 255K and while the relationship between power and temperature is approximately linear on either side of 255K, the current surface temperature of 287K is too far from the reference for the assumption of approximate linearity to be approximately true.

The reason its been so hard for climate science to get this right is that there are many co-dependent and reinforcing errors in the mapping from Bode to the climate system which confuses many into thinking that the model is reasonable. However; without these errors, Bode's model simply can not support the required amplification. Without this support for substantial climate change caused by man, the IPCC and the self serving consensus driven by its reports collapses and to many on that side of the argument, this is an unfathomable consequence, especially given the political ramifications.

In addition to failing to honor the prerequisite assumptions made by Bode, there are other errors regarding how Bode's variables were mapped into climate related variables. This led to an arithmetic error that provided faulty support for a potentially high sensitivity which was never questioned due to confirmation bias. This arithmetic error has to do Hansen's failure to understand the difference between the what Bode calls the feedback fraction and what he calls the feedback factor and this 3 decade old error is still with us today.

The feedback fraction is the fraction of output fed back to the input and is a dimensionless fraction between -1 and 1 spanning a range from 100% negative feedback to 100% positive feedback. The 100% limits arise because you can not feed back more than is coming out of the system in the first place.

Bode defines the feedback factor as the reduction in the open loop gain that arises as the result of feedback. This arises from Bode's gain equation which he states as,

$$E_R = E_0\,\mu/(1 - \mu\beta)$$

Where E_0 is the input to the system (forcing), E_R is the output of the system (the surface temperature), μ is the open loop gain (reference sensitivity, λ_0 per Roe, 2008) and β is the feedback fraction which corresponds to feedback coefficients expressed with units of W/m^2 of feedback per degree K. Bode labels the closed loop gain e^θ which is calculated as $e^\theta = E_R/E_0 = \mu/(1 - \mu\beta)$ and calculates the feedback factor as $e^\theta/\mu = 1/(1 - \mu\beta)$ which is the reduction in μ that results from the application β. The most important aspect of this equation is the μ on both sides of the equals sign. Bode then makes a simplification assuming that $\mu >> 1$ and $\beta < 0$, both of which are true for linear amplifiers and asserts that $\mu\beta$ by itself can also be considered the approximate 'feedback factor'. Modern amplifier design ignores this altogether as the effective μ of modern amplifiers is on the order of many millions and as μ approaches infinity, $\mu/(1 - \mu\beta)$ approaches $-1/\beta$ (the feedback fraction) and the feedback factor becomes infinite.

To adjust the gain equation for COE, the power applied as feedback, $E_r\beta$, must be subtracted from the output since feedback power can not also contribute to the available output. The gain equation that is applicable to the climate becomes,

$$E_R = E_0\,\mu/(1 - \mu\beta) - E_r\beta$$

Climate science incorrectly considers λ_0 times an empirical coefficient, c_1, as the metric to quantify feedback, considers their product to be equivalent to Bode's $\mu\beta$ and calls this the 'feedback factor'. Again, Bode's assumptions were not honored since the climate system μ is very close to 1, and in fact is exactly 1 for an ideal black body, thus the feedback factor would really be $1 - \lambda_0 c_1$. While a compensating error added the 1 back to the equation, it didn't fix the misunderstanding that led to the arithmetic error in the first place.

The arithmetic error arises when to get the units to line up and ostensibly conform to Bode, Roe defines the feedback factor $f = \lambda_0 c_1$ (per Hansen and Schlesinger). If the sum of the input and feedback (the input to the gain block) is J, the output of the gain block is $J*\lambda_0$. Roe's assignment of the feedback factor infers that that $c_1 = f/\lambda_0$. Multiplying the output of the feedback network by f/λ_0 (c_1) produces a feedback term equal to $J\lambda_0 f/\lambda_0$. The λ_0 cancels leading to a feedback term quantified as Jf, where f becomes equivalent to Bode's β when μ is 1 and quantifies both the fraction of output and the fraction of J returned as feedback. The specific arithmetic error is assuming that the open loop gain is both λ_0 and 1 at the same time. This is illustrated in figure 1.

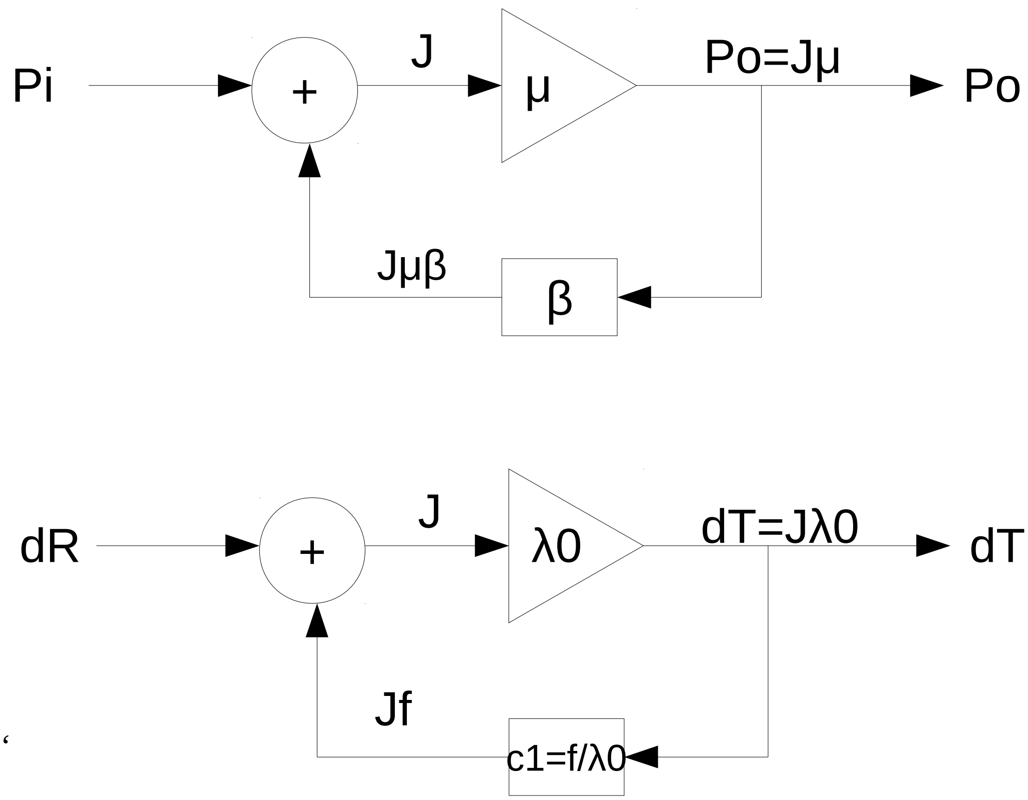

Figure 1

To illustrate the problem further, what μβ is actually quantifying is the post feedback influence of the input of the gain block, J, since the μ term amplifies the input while β takes a fraction of it and returns it as feedback. Conventional climate system feedback assumes that μβ is quantifying the effect feedback has on the output which is only true when μ is 1 and the input and output of the gain block are the same. A more accurate block diagram that represents the consensus climate science feedback model is shown in figure 2.

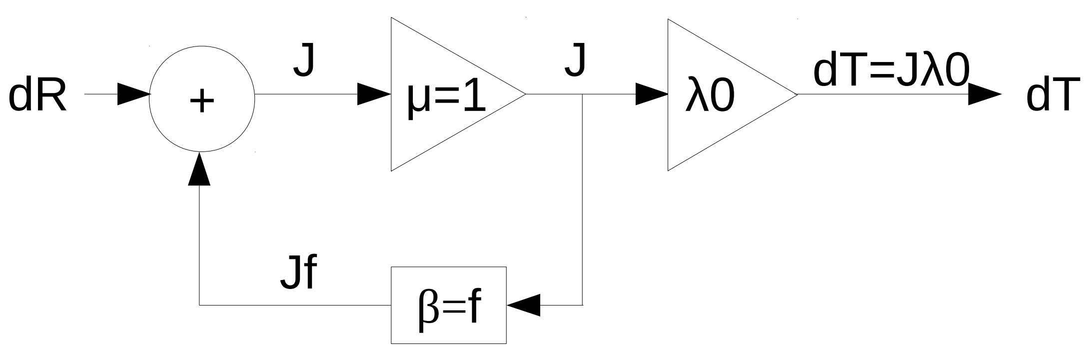

Figure 2

Since the relevant open loop gain is a dimensionless 1, the output of the feedback network must be dimensionally the same as the input otherwise the input plus some fraction of the output can not be summed. The result is that what climate science calls the pre-feedback sensitivity, or the open loop gain, λ_0, is no more than a scale factor applied to the required output in W/m^2 converting it to a change in temperature. In other words, λ_0 is outside of the feedback loop and unaffected by feedback, positive or negative. Calling λ_0 the sensitivity before feedback is incorrect because it has nothing whatsoever to do with the feedback loop being modeled whose gain (sensitivity) is what's being quantified.

This leads to another error which is with Roe's calculation of the system gain, which he considers equivalent to Bode's closed loop gain, e^{θ}. He calculates this as the ratio of two sensitivities. The post feedback sensitivity divided by the zero feedback sensitivity. This implies that feedback amplifies the sensitivity which is not what Bode's model is modeling. It models the amplification applied to an input stimulus to produce an output, where the input is forcing and the output is temperature. While feedback and gain are related, this is a fixed relationship and its the result of this fixed relationship that Bode is modeling. Climate science unfixes this relationship and considers Bode's analysis to apply.

The justification for calculating the closed loop gain in this way comes from Schlesinger, who rationalized that gain could have dimensions because the ratio of gains is dimensionless. Of course, this assumed that the feedback network was modeling a sensitivity input and a sensitivity output, where feedback was modifying the resulting sensitivity while what is actually being modeled is how the surface temperature is affected by incremental forcing and not how feedback is affecting the sensitivity.

Had Hansen and Schlesinger gotten this right in the first place, CAGW would be a footnote warning about jumping to premature conclusions and not an extremely expensive and divisive political issue with either a for or against position in nearly every political platform in the world.

In conclusion, there can be no doubt that the mapping from a Bode feedback system to the climate is irreconcilably broken. Without the ability to claim amplification from large positive feedback, the IPCC looses the only theoretical basis it has for its overstated sensitivity and unless someone invents new physics that transforms 1 W/m^2 of forcing into 4.3 W/m^2 of surface emissions and that doesn't violate Conservation Of Energy, claims of catastrophic effects from CO2 emissions will become as quaint as an Earth centric Universe.

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