The Feedback Fix

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Abstract:

The climate sensitivity required to support catastrophic climate effects caused by CO2 emissions has a theoretical dependence on positive feedback amplifying a small initial effect into a much larger equilibrium effect. The application of feedback to the climate is based on a serious misunderstanding of the linear feedback amplifier analysis referenced as the supporting theory. Feedback analysis will be explained followed by a detailed account of the misunderstandings accompanied with snippets of the papers showing where the errors occurred. It's not useful to deprecate climate feedback analysis without either fixing it or replacing it. An attempt is made to modify the climate feedback model to conform to the preconditions that are currently not being met. This fails owing to the lack of an identifiable origin for any output power in excess of the input forcing. Next, an energy conserving model of the atmosphere's radiant behavior consequential to its self organization by clouds is tested and provides the source of this power as it establishes a quantifiable climate sensitivity that Conservation of Energy requires applies to any and all W/m^2 of forcing. The model also provides mathematical and physical insight into how and why chaotically self organized systems can converge to a deterministic steady state.

Feedback Theory

Feedback is the most misunderstood concept in all of climate science, yet it comprises the theoretical justification for the range of climate sensitivity asserted by the IPCC. To properly understand climate feedback, it's important to understand the actual analysis that was applied to the climate system.

The primary reference cited in all climate feedback papers is Henrick Bode's book on linear feedback amplifier design whose first 2 paragraphs are copied below. The language is dated and esoteric, but it's meaning is precise to those who understand electronic circuits and he assumes that anyone using this analysis does. The asterisk in the introduction refers to contemporaneous references that can supply this background knowledge.

> **1.1** *Introduction*
>
> THE networks to be considered consist of ordinary lumped inductances, resistances, and capacities, together with vacuum tubes. The accessible terminals of the vacuum tubes will be taken as the grid, plate, and cathode. Auxiliary electrodes, such as a suppressor or screen grid, are thus ignored, and the analysis assumes, in effect, that they are grounded to the cathode at signal frequencies. For purposes of discussion the tubes will be replaced by equivalent structures consisting of ordinary circuit elements connected between the accessible terminals, together with a source of current or voltage to represent the amplification of the tube. This ignores such effects as transit time and distributed inductance in the wires inside the tube envelope, which may appear in physical tubes at sufficiently high frequencies.
>
> It will be assumed throughout that all the elements are linear. This chapter is intended principally as a recapitulation of the conventional theory for networks including vacuum tubes in a form which can be used as a foundation for the chapters to follow.*

The preconditions for applying his analysis are underlined. First is that the system requires an implicit power source representing the amplification. For an ordinary amplifier, this is ultimately an electrical outlet or a battery and by convention, power connections are omitted from the active devices in the block diagrams.

Second is that all of its elements are linear implying that the input and output are linearly related to each other across all possible values. For ordinary amplifiers, the input and output are voltages and their constant ratio is the closed loop, or system, gain. If this ratio starts to vary, the amplifier is no longer linear and Bode's analysis no longer applies. For example, when an audio amplifier is overloaded and starts to distort.

The analysis implicitly requires the input and output to be expressed in the same units. Even an amplifier with voltage in and current requires the output current to be passed through a resistor to produce an output voltage that's linear to the output current, a fraction of which can be added to the input voltage as the feedback voltage.

Here's Bode's definition of a feedback network. The input is an explicit time varying signal, P1 is an implicit summation node and P2 is a simple connection.

3.2. *Elementary Theory of Feedback Circuits*

In its simplest form, a feedback amplifier can be regarded as a combination of an ordinary amplifier, or μ circuit, and a passive network, or β circuit, by means of which a portion of the output of the μ circuit can be

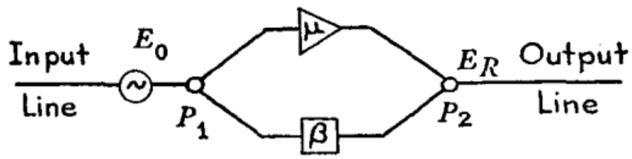
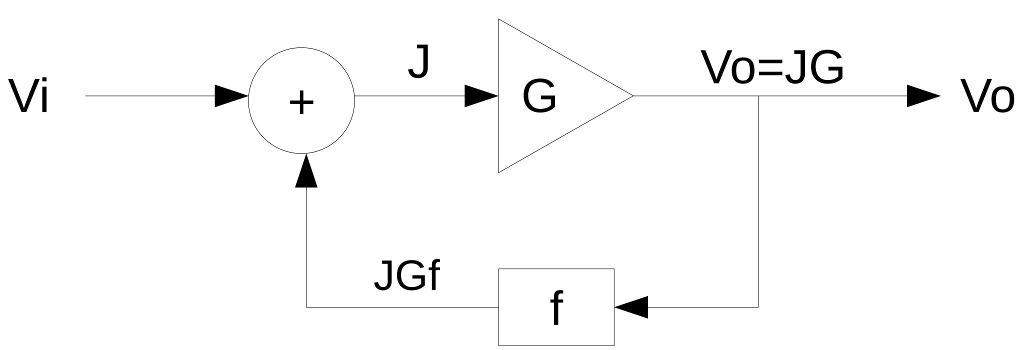


FIG. 3.1

returned to its input. Such a combination is shown by Fig. 3.1. Both the μ and β circuits are, of course, actually four-terminal structures. The circuits are represented by single lines in Fig. 3.1 for simplicity.

The modern representation of a feedback amplifier below is more typical of climate feedback diagrams. The system has an input, **Vi** (**E_0**), an output, **Vo** (**E_R**), an open loop gain, **G** (**μ**), a feedback fraction, **f** (**β**), and a closed loop gain **g** = **Vo/Vi** (**E_R/E_0**). Each of **G**, **f** and **g** must be dimensionless ratios. Bode specifically mentions that both the active **μ** (**G**) and passive **β** (**f**) circuits are completely characterized by the dimensionless ratio of their output voltage to their input voltage. The feedback fraction, **f**, or Bode's **β** circuit, is typically a resistive voltage divider that produces the fraction of output that is then added back to the input. The passive nature of a voltage divider limits feedback to 100% of the output and whether it's positive or negative feedback depends on the phase of the output relative to the input. What negative feedback means is that when the input voltage is rising, the feedback voltage is falling and visa versa. The negative of a sine wave, is another sine wave that's 180 degrees out of phase. Ordinary single stage amplifiers, whether built from a tube or transistor, are inverting amplifiers, where a sine wave input produces a sine wave output of the opposite phase providing negative feedback when a fraction of it is added back to the input.



Bode defines another attribute called the feedback factor, which is the open loop gain times the feedback fraction, **μ*β (G*f)**. This metric is not particularly important except as a demonstration for how a larger open loop gain is better at stabilizing the closed loop gain.

On page 32, Bode defines the systems constraining equation as 3-4 where the system gain, or closed loop gain, becomes **Er/E_0**. He also defines the feedback factor.

$$E_R = \mu E_0 + \mu\beta E_R, \tag{3–3}$$

or in other words

$$E_R = \frac{\mu}{1 - \mu\beta} E_0. \tag{3–4}$$

Without the β circuit, the output voltage would be given by $E_R = \mu E_0$. We therefore have the

Theorem: Feedback reduces the gain of an amplifier by the factor $1 - \mu\beta$.*

The quantity $\mu\beta$ can be called the *feedback factor*.† It evidently repre-

The system gain equation can also be derived from the modern representation of a feedback amplifier by noticing that **J** = **Vi** + **Vo***f** and that **Vo** = **J***G**. Substitute and the result is,

Vo = (Vi + Vo*f)*G

Divide both sides by **Vi** and it becomes,

Vo/Vi = (1 + Vo/Vi*f)*G

Substitute **g** for **Vo/Vi** and we get,

g = (1 + g*f)*G

Now, solve for the closed loop gain, **g**.

g = G/(1 – f*G)

1/g = 1/G – f

Bode's gain equation 3-4 can be rearranged to arrive at the same result in terms of **μ** and **β**.

When **g** goes negative, the amplifier becomes an oscillator. When **G=1**, the system is unconditionally stable for all values of **f < 1** (anything less than 100% positive feedback). If **G** is infinite, then any positive value of **f** is unstable. For most ordinary amplifiers, **G** large enough that the gain equation becomes,

g = -1/f

Feedback is used in amplifiers so that the system gain is largely independent of the open loop gain. The open loop gain varies over a 2:1 range or more owing to the highly variable properties of tubes or transistors and without feedback, any design is much less predictable. Bode states this in a somewhat esoteric manner where as the open loop gain approaches infinity, the system gain is approximately proportional to the **β** circuit loss. He then arrives at this approximation of the system gain equation,

$$E_R/E_0 = -1/\beta$$

The Climate Feedback Model

Feedback came to prominence in climate science in 1984 when James Hansen applied Bode to the climate in this snippet of his paper.

We use procedures and terminology of feedback studies in electronics (Bode, 1945) to help analyze the contributions of different feedback processes. We define the system gain as the ratio of the net feedback portion of the temperature change to the total temperature change

$$g = \frac{\Delta T_{feedbacks}}{\Delta T_{eq}} . \qquad (5)$$

Since

$$\Delta T_{eq} = \Delta T_o + \Delta T_{feedbacks} , \qquad (6)$$

it follows that the relation between the feedback factor and gain is

$$f = \frac{1}{1 - g} . \qquad (7)$$

To use the procedures and terminology of Bode, they must be applied properly. Bode defines the system gain as the ratio of the output to the input which has no resemblance to equation (5). The idea that temperature feedbacks arising from W/m^2 of forcing are primarily quantified by a ΔT is not even physical, since only W/m^2 of feedback can be added to W/m^2 of forcing. Bode adds the feedback to the forcing prior to amplification in order to calculate a new output which is specifically not the sum of a pre-feedback output plus a feedback contribution per equation (6) unless the open loop gain is a dimensionless 1; moreover, Hansen never disclosed, or more likely didn't know, that he implicitly assumed unit open loop gain. He didn't show the work for how equation (7) was derived and ended up flipping the gain and feedback terms. To 'fix' this error, something Michael Schlesinger wrote was published as an inadequately reviewed appendix to an article in a DOE journal (DOE/ER-0237) where he incorrectly derived a somewhat more correct gain equation as,

$\mathbf{R_f = 1/(1 - f)}$

He had **g** (**R_f**) and **f** properly represented, but missed the assumption of unit open loop gain. He justified his conclusion by conflating the feedback factor with the feedback fraction and declared that the feedback factor was the dimensionless constant quantifying the sign and amount of feedback, which is only the case when the open loop gain is 1.

Here is Schlesinger's misunderstanding of feedback from an image of his paper which is word for word the same as the appendix in DOE/ER-0237 that was published 1 year after Hansen's 1984 paper and a few years before Schlesinger's paper appeared in an 1988 publication he edited himself. The lack of review is evident from DOE/ER-0237 and Schlesinger's paper having the same typo referring to page 32 of Bode 1975 instead of Bode 1945.

The effect of the feedback can be characterized on the basis of the ratio of the ΔT_* with feedback to that without feedback. Thus, by Eqs. (11) and (12), we define the feedback gain ratio

$$R_f \equiv \frac{\Delta T_*}{(\Delta T_*)_o} = \frac{G_f}{G_o} = \frac{1}{1 - f} , \qquad (16)$$

where

$$f = G_o F \qquad (17)$$

1945

is the feedback factor (Bode, 1975, p. 32) or, here, simply the feedback.[1] For $f = 0$, $R_f = 1$; hence $(\Delta T_*)_o$ represents the zero-feedback temperature change. Since $0 < R_f < 1$ for $f < 0$, the latter represents negative feedback (see Fig. 2). As negative feedback increases indefinitely, $R_f \to 0$ and $\Delta T_* \to 0$; however, it is important to note that ΔT_* does not change sign as $f \to -\infty$. Since $R_f > 1$ for $0 < f < 1$, the latter represents positive feedback. As positive feedback approaches unity, $R_f \to \infty$ and $\Delta T_* \to \infty$. If the positive feedback could be extended beyond unity, R_f would change sign and approach zero from negative values as $f \to \infty$. Clearly, the region $f > 1$ is physically meaningless.[2] However, as we shall see, one SEBM has estimated de facto such strong positive feedbacks that $f > 1$ and a temperature decrease $\Delta T_* < 0$ was obtained for heating $\Delta Q > 0$!

1 Hansen et al. (1984) call f (their g) the system gain and R_f (their f) the net feedback factor.

The feedback gain ratio, which he defines as the ratio between the closed loop gain and the open loop gain, is a meaningless distraction, since in general, the closed loop gain is largely independent of the open loop gain which is often so large, the feedback gain ratio becomes 0.

However, since the climate feedback model implicitly assumes unit open loop gain, the feedback gain ratio becomes equal to the system's closed loop gain.

From the text and the math, **f** is the feedback factor which is equal to the actual feedback fraction, **F**, times the open loop gain, $\mathbf{G_0}$, while the **f** in the gain equation should then be **F**. He incorrectly defines **f** from 0 to 1 as the dimensionless fraction representing positive feedback and that **f** > 1 (more than 100% positive feedback) is meaningless. The real meaning of **f** > 1 is that the combination of the open loop gain and positive feedback is enough to turn an amplifier into an oscillator and for **f** greater than 1 to indicate more than 100% positive feedback, **F** and **f** must be the same which is only true when $\mathbf{G_0}$ is a dimensionless 1.

Schlesinger also claimed that $\mathbf{G_0}$ could have arbitrary dimensions as long as **F** had complementary dimensions. This fell through to the quantification of the various feedback coefficients, making all of them physically meaningless relative to any measurable metric. Schlesinger's calculation of $\mathbf{R_f}$ would be more correct by acknowledging that $\mathbf{G_0}$ is 1, but is still inconsistent because it would require the output to be W/m^2 and not degrees since you can not add degrees of feedback to W/m^2 of forcing.

Schlesinger assumes that $\mathbf{G_0}$ converts W/m^2 into degrees as he implicitly assumes $\mathbf{G_0}$=1 in order to justify his dimensional $\mathbf{G_0}$! Neither Bode's analysis, nor any other, can be applicable since $\mathbf{G_0}$ can't be a dimensionless 1 and an arbitrary value that converts W/m^2 into degrees at the same time. In principle, Schlesinger's open loop gain, $\mathbf{G_0}$, is the incremental application of the Stefan-Boltzmann Law on the output of a unit gain amplifier where the feedback apparently and impossibly, overrides this law.

His feedback factor error takes $\mathbf{G_0}$ out of the loop, canceling it out of the feedback coefficients and incorrectly justifies applying linear analysis to a non linear relationship. This is one of the primordial failures that the body of bad science supporting climate alarmism is built upon. In this case, Schlesinger wrote a correction to a Hansen paper whose inadequately reviewed and demonstrably wrong analysis was positioned as properly reviewed and settled theoretical support for an effect from CO2 that was large enough to justify the formation of the IPCC and its preordained conclusions about the climate.

The cause of this error and its subsequent declaration as 'settled' was that nobody familiar with Bode's analysis had critically reviewed Schlesinger's work and as a result, confirmation bias prevailed. The peer review of his work that was referenced in AR1 to correct Hansen's more obvious error was limited to editing by Mike MacCracken and Fred Luther who didn't even correct the 1975/1945 typo, although they did clean up his figures. MacCracken is the chief scientist at the Climate Institute, which is a lobbying organization biased towards climate alarmism and tightly connected to Democrat politics. He deferred to Schlesinger as the top feedback expert. Fred Luther was from Lawrence Livermore Labs, was not a feedback expert either, and whose work focused on other climate related concerns.

Schlesinger's flawed application of feedback analysis was eventually formally published by Gerard Roe who made the same errors in his 2009 paper on climate feedback, the most obvious of which is that any feedback network with W/m^2 as input and a temperature output unambiguously violates the linearity precondition, incrementally or not. In the steady state, the only work required is to maintain the temperature by replacing its emissions, which based on the Stefan-Boltzmann Law, is definitely not arbitrarily incrementally linear to temperature.

What follows is the portion of Roe's paper that replicates Schlesinger's feedback factor error, with only the variable names changed. The reason this got past formal peer review was that it referenced and echoed the errors in the Schlesinger paper that was wrongly considered to have been properly reviewed.

In this snippet, Roe specifically misidentifies the feedback factor, ***f***, as the fraction of the output returned to the input whereas per Bode, it's the **β** circuit, or Roe's $\mathbf{c_1}$ that's the actual dimensionless fraction of the output fed back to the input.

the feedback, compared with the reference-system response,

$$G = \frac{\Delta T}{\Delta T_0}. \tag{6}$$

The feedback factor, *f*, is proportional to the fraction of the system output fed back into the input,

$$f = c_1 \lambda_0. \tag{7}$$

In the electrical-engineering literature and the control-systems literature, both c_1 and $c_1\lambda_0$ are referred to as the feedback factor (e.g., Bode 1945, Graeme 1996, Kories & Schmidt-Waller 2003). The above choice is preferred as a nondimensional measure of the feedback. However, it should be borne in mind that, in so choosing, the feedback factor becomes dependent on the reference-system sensitivity parameter.[4]

Combining Equations 5, 6, and 7, it can be shown that

$$G = \frac{\Delta T}{\Delta T_0} = \frac{1}{1 - f}. \tag{8}$$

[4]Hansen et al. (1984) reverses the conventional definition of a feedback factor and a gain, but is thereafter consistent.

As Hansen and Schlesinger did in their earlier feedback papers, Roe cites Bode and as the source of the feedback theory being applied and even got the 1945 reference right.

Roe also characterizes the feedback fraction and the feedback factor as the same thing, which follows from the undeclared and implicit assumption that $\boldsymbol{\lambda_0}$ is a dimensionless 1. To his credit, he recognized that Schlesinger's errors are what enabled $\mathbf{c_1}$ (**f**) and $\boldsymbol{\lambda_0}$ (**G**) to have arbitrary reciprocal dimensions and attempted to explain it by stating that the feedback factor was the preferred dimensionless feedback coefficient over $\mathbf{c_1}$**.** This isn't even an available choice since linearity requires each of ***f***, $\mathbf{c_1}$ and $\boldsymbol{\lambda_0}$ to be constant, dimensionless ratios.

Schlesinger rationalized his analysis by considering approximate linearity around the mean was sufficient to meet the linearity constraint and that the average not accounted for by applying an incremental analysis was the power supply. Approximate linearity around the mean isn't even close to sufficient as the system gain must be the same for all possible inputs and outputs, from 0 to their limits. The average not accounted for by the incremental analysis is not even available to be the power supply, since all of this power is already completely consumed maintaining the average temperature which is also not accounted for.

Can Feedback Theory Be Applied

To address the linearity constraint, the input can be all of the solar forcing and the output can be the radiant surface emissions, both expressed in W/m^2. The radiant surface emissions are the Stefan-Boltzmann emissions of the surface at its average temperature. The complete influence of all non radiant energy fluxes between the atmosphere and the surface, for example, latent heat plus its return the surface, are already accounted for by the average temperature and its radiant emissions and can therefore be considered to have a zero sum influence on the radiant balance and the resulting radiant sensitivity. This is still true even if any or all of the offset of any non radiant energy leaving the surface happened to be radiant.

Best practices modeling considers a change to the system, for example, a change in CO2 concentrations, to be modeled as an equivalent change in solar forcing while keeping the system, that is CO2 concentrations, constant. Considering a change to the system, or a dependence of the system on the input or output as forcing independent of the solar forcing is physically incorrect as neither would have any effect in the absence of solar forcing.

The open loop gain of this model is 1 which without feedback represents an ideal black body where the steady state emissions are equal to the steady state incident energy. The Earth has an average temperature of about 288K with 390 W/m^2 of radiant surface emissions arising from a total incident flux of only 240 W/m^2. The result is a closed loop gain of about 390/240 = 1.62. Each W/m^2 of solar power results in 1.62 W/m^2 of surface emissions and each W/m^2 of the planets emissions requires 1.62 W/m^2 to be emitted by the surface. The IPCC's nominal sensitivity of 0.8C per W/m^2 requires the next W/m^2 of solar forcing to increase the surface emissions by 4.4 W/m^2. This is clearly impossible, as the climate system can't tell one W/m^2 from any other, nor can feedback selectively operate on only the next W/m^2; moreover; all W/m^2, including the next one, are arriving concurrently.

The required feedback for this model is about 38% positive. Any concern that this is positive feedback is a red herring since when the open loop gain is unity, the system is unconditionally stable for all positive feedback less than 100%.

This linear model is still non conforming, as there's no implicit power supply. To compensate, we must conserve energy between the input and output of the gain block. This means that the output power comes from the input and feedback power, thus the output of the gain block can contribute to the system output or become feedback, but not both. The system output will never exceed the input since the boost by feedback will be consumed as the feedback itself.

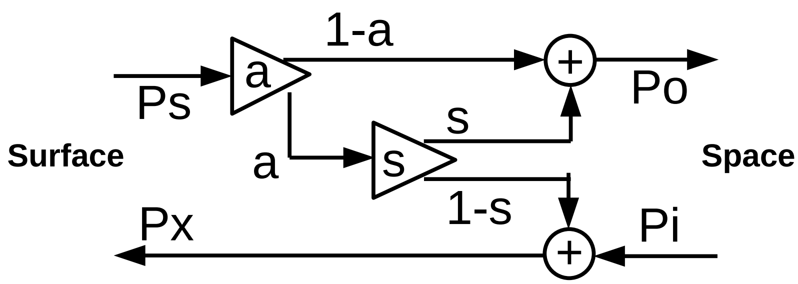
It should be clear that without a power supply, a feedback amplifier can not amplify regardless of the sign or magnitude of its feedback, since without a power supply, there's no amplified output to feed back. The reason Bode's analysis can't be applied to the climate is because it requires active gain, while the climate is a passive system. What distinguishes the two is that the output power of a passive system is dependent only on the input power, while for an active system, the output power is dependent only on the limits of its implicit power supply.

Additionally, Bode assumes zero delay in the feedback path. In the climate system, what is considered the feedback power is delayed and arrives to be combined with new forcing at a later time and is not added to the specific forcing that resulted in the feedback, as is assumed by Bode's analysis.

What System Model Does Work

Consider the following block diagram of a passive radiant model of the Earth's atmosphere. **Po** and **Pi** are the power to and from space, **Ps** is the net surface radiant emissions and **Px** is the power replacing those emissions. We could add another input and output on the surface side of the model representing the non radiant energy leaving the surface and its offsetting return to the surface, but whatever effect that flux has on **a** and **s** is already accounted for by their steady state average values.

The variable **a** represents the fraction of **Ps** that's absorbed by the atmosphere while **1-a** is the fraction not absorbed and sent into space. The variable **s** is the fraction of **a** that's ultimately emitted into space, while **1-s** is the fraction of **a** returned to the surface in order to offset a **Ps** that's greater than **Pi**. The steady state average is defined when the averages of **Ps** are equal to **Px** and **Po** are equal to **Pi**.



The expected and measured value of **s** is about 1/2, with short term deviations of only a few percent on either side. This is expected because it's a first order function of geometry where the area across which the atmosphere emits what it absorbs is twice that over which it's being absorbed. Part of **Pi** is also absorbed and redistributed and can be quantified by the same normalized mathematical transformations that will convert **{Pi,Ps}** into **{Po,Px}**.

The limits on **a** are between 0 and 1. For **a=0**, **Pi=Po=Ps=Px** and is the expected result for an ideal black body. For **a=1**, **Pi=Po**, **Ps=Px** and **Ps=Pi*2** which limits the maximum possible surface sensitivity to 2 W/m^2 of surface emissions per W/m^2 of solar forcing. The IPCC's presumed lower bound is more than 0.4C per W/m^2 which translates into more than 2.2 W/m^2 of surface emissions per W/m^2 of forcing and already exceeds the maximum possible surface sensitivity as limited by this passive, conservation constrained model of the atmosphere. Note that the Venusian **Ps** and **Px** are relative to cloud tops and not the solid surface below since the solid surface and clouds are not coupled together by anything like a hydro cycle.

Earth's **Ps**/**Pi** ratio is analogous to the closed loop gain, **g**, where **Po** = **Ps**/**g** and **Px** = **Pi***g**. The attenuation of **Ps** by **1/g** to produce **Po** is quantifiable as a gray body emitter whose temperature is that of the surface emitting **Ps** and whose emissivity is **1/g** producing an output of **Po**. The surface supports a warmer temperature simply because about half of what's attenuated by clouds and GHG's by the emissivity is returned to the surface to supplement future solar forcing. Some simple math derives the lone free variable in the energy balance, **g,** as a function of **a** given that **s=1/2**.

g = 1/(1 – a/2)

The implication is that for the observed **g** to be 1.62, **a** must be 0.76, meaning that 76% of the radiant energy the surface emits as a consequence of its temperature is absorbed by the atmosphere, half of which is returned to the surface at a later time in order to sustain more future emissions than future solar forcing can achieve on its own, while the remaining half is emitted into space to supplement what is passed directly to space in order to offset all of the past incident solar energy. Note the correspondence of this to the feedback gain equation, where given the assumption of unit open loop gain, **a/2** is the amount of positive feedback required to achieve a system gain of 1.62. Relative to Schlesinger's analysis, **f = F = a/2** and is equal to the same 38% positive feedback required by the linearized feedback model.

What the climate feedback model is attempting to calculate can be derived by converting the surface emissions, **Ps,** to a temperature, **Ts**, by inverting the Stefan-Boltzmann Law.

$$Ts = (Ps/\sigma)^{0.25}$$

To calculate the steady state sensitivity, substitute **Pi*g** for **Ps** and then differentiate **Ts** with respect to **Pi** to arrive at the steady state surface emissions sensitivity to solar forcing which is exactly what the climate feedback model is trying to predict, where **dTs** is the incremental output of the model when **dPi** is the incremental input.

$$dTs/dPi = g/(4*\sigma*T^3)$$

Substituting 288K for **T,** 1.62 for **g** and the calculated sensitivity of the surface temperature to changes in W/m^2 of total forcing becomes **0.3K per** $\mathbf{W/m^2}$. This is the one and only relevant sensitivity metric since 1 Watt is 1 Joule per second, Joules are interchangeable and Joules are the units of work. While the work required to change the temperature is linear to the change in temperature, the work required to maintain the temperature is proportional to the temperature raised to the fourth power which is all that matters in the steady state since the temperature has already changed.

Hansen mentions this sensitivity analysis in an appendix to his paper and calculates the same value using an effective emissivity of **1/g**, but incorrectly considers the calculated gain to be a pre-feedback result. This is another failure in the logic supporting the application of feedback analysis to the climate. This calculation is post feedback since all feedback like effects have already had their full effect on all of the solar forcing that results in the steady state surface temperature, its emissions and sensitivity. Feedback just can't tell one Joule from another so that it only applies to the next W/m^2 and not all of the others.

The Math

A 2x2 **R**adiant **T**ransform **M**atrix (**RTM**) converting **{Pi,Ps}** into **{Po,Px}** quantifies math for the passive models bidirectional radiant transfer function and can be represented by a matrix equation which will be called the **A**tmospheric **R**adiant **T**ransform (**ART**).

$$\begin{vmatrix} Po \\ Px \end{vmatrix} = \begin{vmatrix} a00 & a01 \\ a10 & a11 \end{vmatrix} * \begin{vmatrix} Pi \\ Ps \end{vmatrix}$$

When the **RTM** is the identify matrix **{1, 0}, {0, 1},** it models the steady state condition where **Po=Pi** and **Px=Ps**. When it's the gain matrix **{0, 1/g}, {g, 0},** it models the other steady state condition where **Ps=Po/g** and **Px=Pi*g**.

A third **RTM** represents the normalized radiant transform where both outputs are functions of both inputs. This transformation matrix is given by **{-1/g, 1}, {1, 1-1/g}**. Applying these three **RTM**'s to the **ART** will produce the same results for any valid value of **g**; moreover, any average behavior of the atmosphere must conform to all three equations. The first **RTM** is the COE constraint and in a linear algebra context, the second **RTM** represents an eigenvector of the third, more general transformation.

The instantaneous **RTM** is a strong function of cloud coverage since total absorption is also a strong function of clouds. For cloudy skies, **a** and thus **g** are above average, while for clear skies, both are below. Clouds self organize the atmosphere into an optimum configuration that drives **a** and **g** to some goal between cloudy and clear. The goal of all self organization is to optimize the use of energy, which for the climate means maximizing surface warmth given the available solar energy. This means keeping the average **g** as constant as possible since changing the average **g** takes work that's not otherwise available to warm the surface.

The effects of albedo can't be discounted, although the **ART** only quantifies the relationship between the post albedo solar energy and the radiant emissions of the surface. To the extent that something changes the albedo as a function of temperature or even CO2, it will warm or cool a bit more, but this is more properly represented as a change in forcing and not feedback. Besides, there's not a lot of room for average ice to drop much lower than it already is, since it always reforms in the winter. We also know that polar ice survives much warmer global temperatures then exist today because the ice cores are still here to tell us.

The allowable values of **a** are between 0 and 1 corresponding to values of **g** between 1 and 2, but a constant average of **g=1.62** seems to be preferred, so why is that since any value of **g** satisfies all 3 **RTM's** and a radiant balance can be achieved for all values of **a, s** and/or **g**?

There must be another constraint for quantifying the behavior leading to this preferred value of **g** since even its local monthly and yearly averages are unusually consistent as compared to other climate metrics. When attempting to calculate a solution for the radiant balance, there's always one free variable which implies that any radiant balance is possible, yet one average consistently emerges. The hypothesis here is that this average emerges from chaotic self organization and that the strange attractor of the chaos comprises the last constraint required to solve the radiant balance and subsequent sensitivity.

Clouds chaotically vary the instantaneous **RTM** and there are an infinite number of combinations of coefficients that will be consistent with any single value of the free variable **g.** Most values of **g** have only a single normalized **RTM** whose coefficients are functions of **g** leading to a single slope of its behavior as it approaches its value from either side.

There are a few values of **g** that have an infinite number of possible normalized **RTM'**s each of which has a unique slope as **g** deviates around its nominal value. These values of **g** are fractalized by recursively substituting the **g'**s in the **RTM's** with functions of **g** and powers of **g** that are equal to **g** without adding any free variables. Values of **g** with this property are solutions to the equation $\mathbf{g^2 +/- g = N}$ where **N** is an integer. This is the strange attractor which may also be recognized as related to the Mandelbrot generating polynomial.

The positive real values of **g** that satisfy the strange attractor and are between the limits of 1 and 2 that bound **g** are 1.0, 1.30278, 1.56155, 1.61803, 1.79129 and 2.0. The result is that these values of **g** are much more likely to emerge from chaotically driven self organization since when **g** is close to one of these fractalized values, a sequence of chaotic **RTM**'s is far more likely to match the slope of a matching incremental behavior driving **g** to that value.

The values 1.0, 1.61803 and 2.0 are solutions of $g^2 - g = N$, for **N = 0, 1, 2** and represent the 2 limits and a mid value that just happens to be within 1% of the measured average value of **g**. The others are solutions to $g^2 + g = N$, for **N=3, 4, 5** and represents other stable states that the system could potentially converge to. For the climate to transition to a larger **g**, it requires nearly 100% cloud coverage, so while the surface temperature will be warmer given the same amount of solar forcing, the increased albedo will significantly decrease the total solar input power. Although, a value of **g**=1.56155 and a colder climate may emerge during ice ages when the albedo effects of clouds are obscured by surface reflection.

The climate system is a multi-stable system with 6 potentially stable values of **g**. Which one it converges to depends on which is closest to the center of the chaotically variable range of **a**. The current range observed for Earth is from a minimum of about 45% for clear skies to a maximum of about 90% for the cloudiest skies, corresponding to a range of instantaneous **g** of from about 1.3 to 1.8 making the only two values that can reasonably emerge as an average 1.56155 and 1.61803 indicating that the climate system may be bi-stable, is currently in the warm state and the only possible 'tipping point' will be towards a cooler climate.

The value of **g**=1.61803 might also be recognized as the limit of the ratio of sequential elements in the Fibonacci series and as the golden ratio. That this ratio emerges from fractal math quantifying chaotic self organization and applies to the climate is both profound and testable.

Conclusion

The application of positive feedback defies conservation laws by amplifying the climate sensitivity without bound and it's absurd that this is accepted as 'settled' science. The errors that led to this faulty conclusion must be corrected. When proper science is applied to the climate system, it becomes obvious that climate alarmism is a waste of time, money and intellectual capital that's destined to destroy modern society by causing It to waste trillions of dollars attempting to fix a climate that does not need to be fixed all under the guise of a fake greater good that's continually reinforced with emotional manipulation by the media.

The only energy related danger we face is running out of oil and the only viable solution is nuclear. Transient sources like solar, wind and batteries have a place, but that place is definitely not as a base load source of power. Over committing to renewable energy is a clear and present danger whose detrimental effects are already being felt and will only get worse as the Biden administration is destined to commit trillions of dollars to green virtue signaling whose only tangible effect will be the decimation of our economy in order to make climate alarmists feel better about themselves.

The biggest existential threat we face today is climate alarmism, which the Biden administration obsessively thinks is its most supportable cause. Their justification is based on science that's so wrong, and in so many ways, it's an embarrassment to legitimate science. Unless climate science can be repaired, we face a dark, cold and oppressive future.

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