

# Why 0.8C per W/m<sup>2</sup> Is Impossible

By George White - 8/1/15

## Abstract

First principles physics are applied to derive the sensitivity to forcing for an ideal gray body. The Earth's measured average temperature, input power and output power are mapped to this model which demonstrates that as a predictor of the planet's LTE response to change, the ideal gray body maps well to the observed data. Satellite data is presented which confirms that over a wide range of temperatures and emissions, the planet behaves nearly exactly like an ideal gray body implying that with near absolute certainty, the post feedback, long term sensitivity of the Earth's surface temperature to forcing has an upper bound prescribed by physical laws that is well below the lower bound of the IPCC's stated range of 0.8C +/- 0.4C per W/m<sup>2</sup>.

Derive an exact representation for the sensitivity of an ideal gray body as a function of state whose LTE temperature is  $T$ , whose average input and output power is  $P$  and whose average emissivity is  $\epsilon$ . The Stefan-Boltzmann Law expresses  $P$  in terms of  $T$  and  $\epsilon$  where  $\sigma$  is the Stefan-Boltzmann constant,

$$1) P(T, \epsilon) = \epsilon\sigma T^4$$

Define the sensitivity,  $\lambda(P, T, \epsilon)$ , as the LTE change in  $T$  consequential to a change in  $P$ , at some  $\epsilon$ .

$$2) \lambda(P, T, \epsilon) = dT/dP$$

Solve equation 1) for  $T$  and differentiate with respect to  $P$ .

$$3) T = (P/(\epsilon\sigma))^{0.25}$$

$$4) u = (P/(\epsilon\sigma))^{0.5}$$

$$5) T = u^{0.5}$$

$$6) dT/dP = 0.5/u^{0.5} * du/dP$$

$$7) dT/dP = 0.5/u^{0.5} * 0.5/u * 1/(\epsilon\sigma)$$

Combining equations 2), 4) and 7) results in an expression for the sensitivity as a function of  $P$  and  $\epsilon$ .

$$8) \lambda(P, \epsilon) = (P^{-0.75} * (\epsilon\sigma)^{-0.25}) / 4$$

Combining equations 2), 5) and 7) results in an expression for the sensitivity as a function of  $T$  and  $\epsilon$ .

$$9) \lambda(T, \epsilon) = T^{-3} / (4\epsilon\sigma)$$

Equation 8) can be rewritten as the ideal black body sensitivity,  $\lambda P(P)$ , scaled by a dimensionless factor  $\lambda P\epsilon(\epsilon)$ , where the scale factor at  $\epsilon = 1$  is one.

$$10) \lambda(P, \epsilon) = \lambda P(P) * \lambda P\epsilon(\epsilon)$$

$$11) \lambda P(P) = (P^{-0.75} * \sigma^{-0.25}) / 4$$

$$12) \lambda P\epsilon(\epsilon) = \epsilon^{-0.25}$$

Similarly, equation 9) can be rewritten as the ideal black body sensitivity scaled by a function of  $\epsilon$ .

$$13) \lambda(T, \epsilon) = \lambda T(T) * \lambda T\epsilon(\epsilon)$$

$$14) \lambda T(T) = T^{-3} / (4\sigma)$$

$$15) \lambda T\epsilon(\epsilon) = \epsilon^{-1}$$

Equations 8) through 15) show that for a gray body, the sensitivity is not independent of  $P$ ,  $T$  or  $\epsilon$ , but wholly dependent on the relative values of these variables, moreover; *given known values for any two of  $P$ ,  $T$  or  $\epsilon$ , only one value for the remaining variable works such that  $\lambda(P, T, \epsilon) = \lambda(T, \epsilon) = \lambda(P, \epsilon)$ .*

Applying this model to the Earth,  $P$  is the average power arriving to and radiating from the planet ( $240 \text{ W/m}^2 @ 255\text{K}$ ),  $T$  is the average temperature of the surface ( $288\text{K}$ ) and if the planet behaves like a gray body,  $\epsilon$  is the ratio between  $P$  and the Planck emissions of an ideal black body at temperature  $T$  ( $390 \text{ W/m}^2 @ 288\text{K}$ ), which is about 0.615. Using these values, the sensitivity

calculated by both equations 8) and 9) is 0.300 C per W/m<sup>2</sup>. This passes the first test which is that if  $\epsilon$  was inconsistent with the ideal gray body model, equations 8) and 9) would produce conflicting results, moreover; from equation 9), the required  $\epsilon$  for a sensitivity of 0.8 C per W/m<sup>2</sup> @ 288K is about 0.23.

Whether or not this is proof disputing a sensitivity greater than 0.4C per W/m<sup>2</sup> ( $\epsilon = 0.46$ ) depends only on how close the planets response is to that of an ideal gray body. Figure 1 illustrates equation 1) at 4 different emissivities (0.4, 0.6, 0.8 and 1.0) along with measured monthly average emissions by the planet plotted along the X axis (P) vs. the monthly average surface temperature along the Y axis (T).

Figure 1

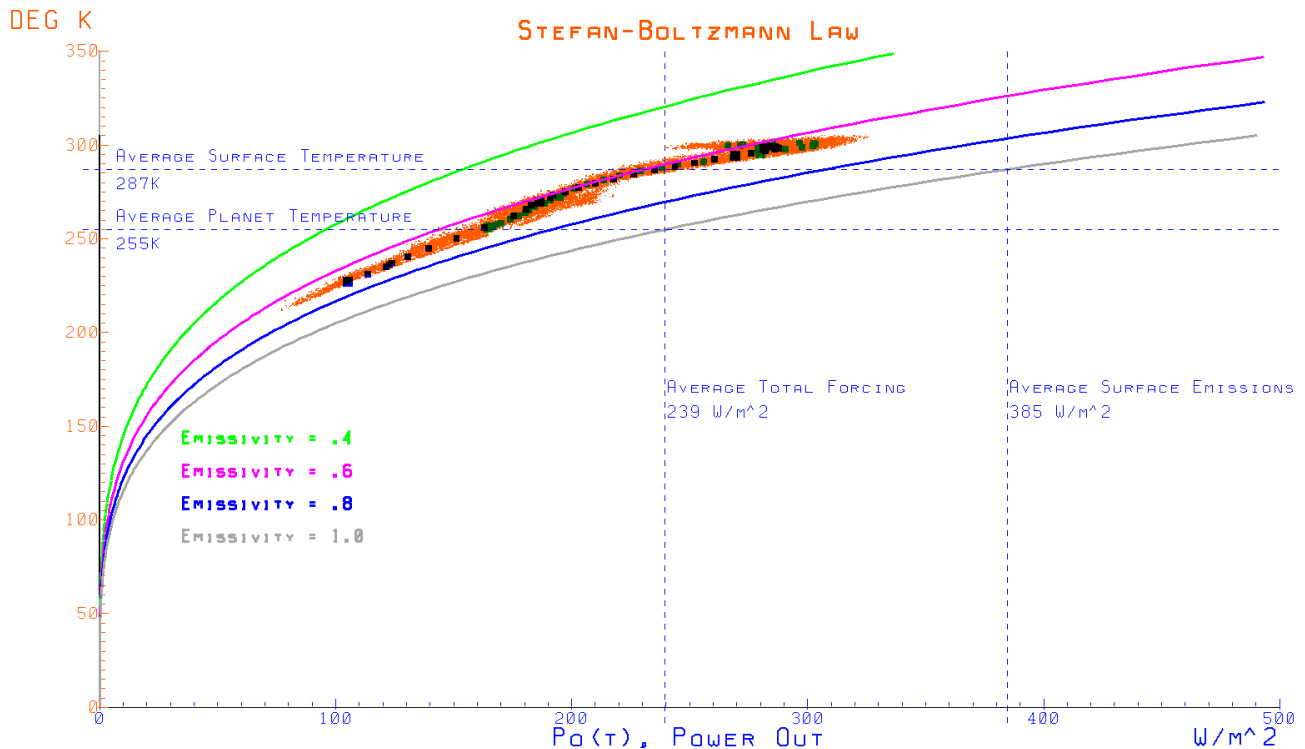


Figure 1 shows that the Earth behaves almost exactly like an ideal gray body whose average emissivity is a little over 0.6. The data plotted aggregates over 100 billion remotely sensed measurements, where each of the 23K small dots is the intersection of the average of all pixels in a 2.5 degree slice of latitude during one month. The 72 larger dots are the per-slice averages across nearly 3 decades of monthly averages. The average slope of the dots at the intersection of the average surface temperature and the average accumulated forcing is about 0.3, as the gray body theory suggests. *The immutability of the Stefan-Boltzmann Law is revealed by the Earth's emissions relative to the surface temperature which are the result of a complex and constantly varying mixture of cloudy and clear skies, yet the macroscopic LTE behavior of the system converges to that of an ideal gray body.*

The data originates from the ISCCP project at GISS which produced an aggregation of weather satellite imagery with full daily coverage of the entire planets surface, most of which is sampled 8 times per day with redundancy from multiple satellites. The output emissions of the planet as a function of time and space are directly measured by the satellite sensors and the surface temperatures are derived by ISCCP to fit the more easily predicted surface temperatures under clear sky conditions. Unlike the GISS surface temperature record, there are no local 'adjustments' to the data, none the less, if any adjustments are introduced they will have little influence on the basic shape of the Earth's response to forcing.

## What About Feedback?

Feedback is the most enigmatic part of climate science and takes on powerful properties as it provides the wiggle room to support a high sensitivity. It's often incorrectly asserted that the ideal gray body model is the 'zero feedback model'. In fact, only the unit emissivity gray body, or the ideal black body, shown as the gray line in Figure 1, represents the behavior of the zero feedback, or unit gain, system.

Control theory tells us that open loop gain ( $G_c$ ) and feedback ( $f$ ) can be traded off against each other to achieve a specific closed loop gain ( $G_c$ ) based on the gain equation,  $1/G_o = 1/G_c + f$ . The measured closed loop gain at the Earth's surface is about 1.6 ( $1/\epsilon$ ) where each of the  $W/m^2$  of accumulated forcing results in  $1.6 W/m^2$  of surface emissions. For the unit open loop gain assumed by the IPCC ( $G_o = 1.0$ ), 37.5% positive feedback ( $f = 0.375$ ) is required to achieve the measured  $G_c$ . If  $G_o$  is 1.6, no feedback is required and if  $G_o$  is 2.0, 12.5% negative feedback is required. Only whether feedback is incrementally increasing or decreasing really matters. Whether the net feedback is positive or negative is a distraction arising from a naive understanding of the relationship between feedback and gain leading to the deceptive positioning of positive feedback as scary and negative feedback as safe.

As the net feedback becomes more positive, the closed loop gain increases and the response moves towards a contour of lower emissivity. Going the other way, the response moves towards a contour of higher emissivity. What remains the same, is that the closed loop gain,  $G_c$ , (surface emissions as a fraction of input power) is the reciprocal of the equivalent emissivity,  $\epsilon$ , and since the only quantifiable effect feedback can have is on  $G_c$ , it must similarly affect  $\epsilon$ .

The effect of incrementally variable feedback is also evident in Figure 1. As the temperature approaches 273K, water vapor and ice feedback increases and the trend is towards a lower emissivity. A transiently larger sensitivity arises as the accumulated prior forcing becomes affected by the newly emerging feedbacks. Extrapolating this towards zero, GHG gases precipitate out of the atmosphere and the response steps closer and closer to the unit gain (unit emissivity/zero feedback) response.

Many estimates of contemporary and palaeo climate sensitivity reflect the transient increase as feedback increases around 273K and incorrectly extrapolate this to the entire Earth's surface illustrating one of the many pitfalls of overzealous homogenization. Even more distracting is that when water vapor is considered by itself, the sensitivity is even higher since the effect observed in the data includes partially offsetting negative feedbacks that are unavoidable, unacknowledged and unaccounted for.

The incremental slope at the current average state (288K,  $240 W/m^2$ ) is the most relevant to any incremental change and by trending towards a higher emissivity, it indicates a small net decrease in the feedback and the actual sensitivity at 288K must be even less than 0.3 C per  $W/m^2$ .

## Conclusion

The theoretical model for the sensitivity of an ideal gray body is irrefutable. The only possible conclusion is that for the Earth's surface sensitivity to be as high as the IPCC asserts, it's behavior must deviate significantly from a gray body and violate the Stefan-Boltzmann Law. The evidence that the Earth's behavior is like an ideal gray body is quite strong and sets a very high bar for any evidence that might dispute the conclusion that with near absolute certainty, the actual sensitivity of the LTE surface temperature to forcing must be less than the IPCC's declared lower bound of 0.4C per  $W/m^2$ .

References:

IPCC reports, definitions of forcing, feedback and sensitivity AR5  
IPCC reports, IPCC, 2013, p.1033 and p. 8SM-7] (AR5)

Rossow, W.B., A.W. Walker, D.E. Beuschel, and M.D. Roiter, 1996: International Satellite Cloud Climatology Project (ISCCP) Documentation of New Cloud Datasets. WMO/TD-No. 737, World Meteorological Organization, 115 pp.

Schiffer, R.A., and Rossow, W.B., 1983: The International Satellite Cloud Climatology Project (ISCCP): The First Project of the World Climate Research Programme. Bull. Amer. Meteor. Soc., 64, 779-784.

Bode H, Network Analysis and Feedback Amplifier Design  
gain equation, page 32 equation 3-3