Transmuting Chaos into Gold

by
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The golden ratio and the negative of its reciprocal are the roots of the golden polynomial given as,

1) $g^2 - g - 1 = 0$

If the behavior of a chaotically variable system can be cast into a polynomial of this form, the emergence of the golden ratio can be predicted.

Consider a bidirectional transfer functions for a 4 port system with 2 inputs and 2 outputs.   The required characteristics of this system are that it be causal with outputs dependent only on its inputs. The generalized transfer function represented in vector form is shown by equation 2).

```
2) | o0 | = | i0 | * | a00 a01 |
   | o1 |   | i1 |   | a10 a11 |
```

If each of i0/o0 and i1/o1 are considered paired ports where in the steady state, i0=o0 and i1=o1, this conservation constraint can be expressed with the identity matrix as shown in equation 3).

```
3) | o0 | = | i0 | * | 1 0 |
   | o1 |   | i1 |   | 0 1 |
```

The 2 paired ports need not be equal to each other, but to the extent that the output from one pair is different than the input of the other, that difference must be offset by the complementary ports. If the ‘gain’ of the system is G, where o1 = G*i0, then another steady state requirement is that o0 = i1/G. This cross conservation constraint can be expressed in vector form as equation 4).

```
4) | o0 | = | i0 | * | 0 1/G |
   | o1 |   | i1 |   | G  0  |
```

Equations 3) and 4) represent the eigenvectors an arbitrary transformation specified by equation 1) must have to conform to the required constraints. In general, each output can be dependent on both inputs, so a normalized form that takes this into account is expressed as equation 5).

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5) | o0 | = | i0 | * | 1-G   1    |
   | o1 |   | i1 |   |  1  1-1/G |
```

For such a system to be in a steady state equilibrium, all 3 of these vector equations must be true for all valid values of G. If i0 is the only source of input to the system and i1 is the return of o1, there’s a system of 3 equations, 1 known variable (i0) and 4 unknowns (o0, i1, o1 and G). Given an arbitrary value of G and i0, the values of the other 3 variables can be easily computed.

Restate equation 5) by considering the coefficients are unknowns, producing equation 6).

```
6) | o0 | = | i0 | * | a0 1 |
   | o1 |   | i1 |   | 1 a1 |
```

Multiplying this out gets,

```
6a) o0 = a0*i0 + i1
    o1 = i0 + a1*i1
```

Substitute with equation 3) where o0=i0 and i1=o1 and restate in terms of i0 and o1.

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6b) i0 = a0*i0 + o1
    o1 = i0 + a1*o1
```

Calculate G as o1/i0, per equation 4).

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   G = o1/i0 = (i0 + a1*o1) / (a0*i0 + o1)
```

Divide RHS by i0/i0 and substitute equation 4) where o1/i0 = G and restate in terms of G.

$$G = (1 + a1*G) / (a0 + G)$$
$$a0*G + G^2 = 1 + a1*G$$

7) $G^2 + (a0 - a1)G - 1 = 0$

If a0 and a1 are replaced with 1-G and 1-1/G, this becomes,

$$G^2 + (1 - G - 1 + 1/G)G - 1 = 0$$
$$G^2 + (G - G^2 - G + 1) - 1 = 0$$
$$G^2 + (G^2 + 1) - 1 = 0$$
$$0 = 0$$

From this result, there's no apparent preferred value of G which means that any value is consistent with the constraining equations 3), 4) and 5), as expected.

However; if (a0 – a1) is equal to -1, instead of 1/G - G, then equation 7) becomes equation 1) whose roots are golden and a preferred value of G suddenly emerges. One way to do this is to replace a0 with -1/G, such that a0 – a1 = -1. Another is to replace a1 with 2 – G, such that a0 – a1 = -1. In both cases, the absolute and incremental behavior is consistent with the 3 constraining equations, providing G is the golden ratio, g. This can be shown by recognizing that when G=g,

8) $g = 1 + 1/g$
9) $g^2 = 1 + g$

When 8) is solved for -1/g, it becomes 1 – g which is the value of a0 in equation 5). A similar analysis holds for the second value of a1. Given the unique golden relationships between g^2, g and 1/g, there's an infinite number of functions of g for both a0 and a1 that are consistent with the constraining equations when G is the golden ratio, moreover; each of the infinite possibilities for a0 can be combined with any of the infinite possibilities for a1. Any possible static or dynamic relationship between i0, i1, o0 and o1 can be quantified by one of the infinite possible transfer functions which prefers a golden value of g. The probability of coincidentally being consistent with the constraining equations for any value of G other than g, is virtually zero.

Some of the infinite possible values of a0 and a1 as a function of g with golden solutions are shown in table 1.

Table 1

a0	a1
1 - g	1 - 1/g
-1/g	2 - g
2 - g^2	1/g^2
1/g^2 - 1	3 - g^2
-2/(g^3 - 1)	1 - 2/(g^3 - 1)
-1/(g^3 - g^2)	1 - 1/(g^3 - g^2)
-(1/g^4 + 1/g^3)	3 - g^4 - g^3
(g^4 - g^3)-3	1 – (1/g^4 + 1/g^3)

Any combination of a0 and a1 values has the identical behavior when g is the golden ratio, while any other value of g is supportable only when a0 = 1-g and a1 = 1-1/g. As g deviates from golden, each combination of a0 and a1 results in a unique response as it restores g to golden. Collectively, these values of a0 and a1 will be referred to as the chaotic golden transform where the constraint equations comprises the attractor of the chaos.

Any system of N pairs of independently conserved input and output ports can be reduced to the 2x2 transform provided that a value of g can be identified which quantifies the relationship between any 2 pairs of ports.

If the chaos is varying the transfer function, effectively varying a0 and a1, the system is far more likely to be driven by a golden combination of a0 and a1 then by any other. Once the system achieves a golden state and starts to deviate from it, there are so many possible behaviors quantified by golden transfer functions to bring it back, it's likely that whatever the behavior is, one of these functions will match and return it to golden.

It's a matter of probabilities. If the transfer function is chaotically varying, all values among the infinite possible values of G, other than g, have only one quantifiable behavior that can achieve that value of G, while G=g has an infinite squared number of transfer functions describing possible behaviors that can achieve that goal. The only precondition is that the chaos must span values of a0 and a1 that can manifest G=g, otherwise, any G is still possible, although it will not have a preferred value and will bounce around chaotically.

In this case, the golden ratio arises as the result of 2 independent conservation requirements with an implicit conservation requirement connecting the two. It’s conceivable that all occurrences of the golden ratio in nature may have this kind of analysis as their mathematical basis. Applications may include biology, finance, galaxy formation, cloud behavior and anywhere else where this ratio emerges out of chaos. Such behavior is not random, but whatever causal behavior is providing the chaos, it must be quantifiable by one of the infinite functions of a0 and a1 that will lead towards golden solutions.